

Dynamical equations for mesons and baryons in large N_c QCD

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Abstract

New equations are derived for meson and baryon Green's functions in a systematic method using the QCD Lagrangian and large N_c approximation as a starting point. The equations are shown to contain both confinement dynamics and chiral symmetry breaking. Linear Regge trajectories are demonstrated for the solutions.

1. Introduction

The QCD dynamics of $q\bar{q}$ and $3q$ systems is governed by two basic phenomena: confinement and chiral symmetry breaking (CSB), which should be treated in a fully relativistically covariant way. Confinement is usually introduced for static quarks via the area law of the Wilson loop [1] or equivalently through the field correlators in the Field Correlator Method (FCM) [2,3].

For spinless quarks, or neglecting spin-dependent mass corrections, one can envisage a self-consistent method which treats confinement as the area law also for light quarks in a relativistically covariant way. Such method was introduced originally in [4] for mesons, in [5] for baryons, and in [6] for heavy-light mesons, and later on in [7] an improvement of the method was done taking into account dynamical degrees of freedom of the QCD string, which naturally appears due to the area law.

As a result Regge trajectories have been found in [7] with the correct string slope $(2\pi\sigma)^{-1}$.

Spin corrections have been considered in [8] for heavy mesons and in [6] for heavy-light ones, while baryon Regge trajectories have been found in [5], for review see [9].

In all cases the basic formalism was FCM and the Feynman-Schwinger (or world-line) path integral representation [3,10,11] which is well suited for relativistic quarks when spin is considered as a perturbation.

The main difficulty which was always present in the method, was this perturbative treatment of spin degrees of freedom (which is incorrect, e.g., for pion) and absence of spontaneous CSB effects in general [12].

Recently a new type of formalism was suggested to treat simultaneously confinement and CSB and a new nonlinear equation was derived for a light quark in the field of heavy antiquark [13].

This equation derived directly from QCD Lagrangian was found to produce linear confinement and CSB for the light quark [13,14] and the explicit form of the effective quark mass operator $M(x, y)$ was defined obeying both these properties.

Since the method of [13] is quite general and allows to treat also multi-quark systems, we apply it here to the $q\bar{q}$ and $3q$ systems, and find dynamical equations for them, which contain confinement and CSB. To make our equations treatable, we systematically exploit the large N_c limit, and mostly confine ourselves to the simplest field correlators – the so-called Gaussian approximation; it was shown in [13] that the sum over all correlators does not change the qualitative results, however the kernel of equations becomes much more complicated.

The paper is organized as follows.

In Section 2 the general effective quark Lagrangian from the standard QCD Lagrangian is obtained by integrating out gluonic degrees of freedom, and the nonlinear equation for the single quark propagator S (attached to the string in a gauge-invariant way) is derived.

Section 3 is devoted to the $q\bar{q}$ Green's function, which can be expressed as an integral of product of S for quark and antiquark. Resulting equation is studied both in the differential form and in the integral form where again the Feynman–Schwinger representation leads to the effective Hamiltonian for the $q\bar{q}$ system, discussed in Appendix.

A similar procedure is accomplished for the baryon Green's function in Section 4. Inclusion of chiral degrees of freedom is given in Section 5, where an effective chiral Lagrangian is derived for bosons. Discussion of results and comparison to other methods is done in Conclusion.

2. Effective quark Lagrangian

As was discussed in the previous section, one can obtain effective quark Lagrangian by averaging over background gluonic fields. We shall repeat this procedure following [13] now paying special attention to the dependence on the contour in the definition of contour gauge, and introducing the operation of averaging over contour manifold. The QCD partition function for quarks and gluons can be written as

$$Z = \int DAD\psi D\psi^+ \exp[L_0 + L_1 + L_{\text{int}}] \quad (1)$$

where we are using Euclidean metric and define

$$L_0 = -\frac{1}{4} \int d^4x (F_{\mu\nu}^a)^2, \quad (2)$$

$$L_1 = -i \int {}^f\psi^+(x) (\hat{\partial} + m_f) {}^f\psi(x) d^4x, \quad (3)$$

$$L_{\text{int}} = \int {}^f\psi^+(x) g \hat{A}(x) {}^f\psi(x) d^4x. \quad (4)$$

Here and in what follows ${}^f\psi_{a\alpha}$ denotes quark operator with flavour f , color a and bispinor index α .

To express $A_\mu(x)$ through $F_{\mu\nu}$ one can use the generalized Fock–Schwinger gauge [15] with the contour $C(x)$ from the point x to x_0 , which can also lie at infinity,

$$A_\mu(x) = \int_c F_{\lambda\beta}(z) \frac{\partial z_\beta(s, x)}{\partial x_\mu} \frac{\partial z_\lambda}{\partial s} ds. \quad (5)$$

Now one can integrate out gluonic field $A_\mu(x)$, and introduce an arbitrary integration over the set of contours $C(x)$ with the weight $D_\kappa(C)$, since Z is gauge invariant and does not depend on contours $C(x)$. One obtains

$$Z = \int D\kappa(C) D\psi D\psi^+ \exp\{L_1 + L_{\text{eff}}\} \quad (6)$$

where the effective quark Lagrangian L_{eff} is defined as

$$\exp L_{\text{eff}} = \langle \exp \int {}^f\psi^+ \hat{A} {}^f\psi d^4x \rangle_A. \quad (7)$$

Using cluster expansion L_{eff} can be written as an infinite sum containing averages $\langle (\hat{A})^k \rangle_A$. At this point one can exploit the Gaussian approximation, neglecting all correlators $\langle (\hat{A})^k \rangle$ for degrees higher than $k = 2$. Numerical accuracy of this approximation was discussed in [9,16] and tested in [17]. One expects that for static quarks corrections to Gaussian approximation amount to less than 10%.

The resulting effective Lagrangian is quartic in ψ ,

$$L_{\text{eff}}^{(4)} = \frac{1}{2N_c} \int d^4x d^4y {}^f\psi_{a\alpha}^+(x) {}^f\psi_{b\beta}(x) {}^g\psi_{b\gamma}^+(y) {}^g\psi_{a\delta}(y) J_{\alpha\beta;\gamma\delta}(x, y) + O(\psi^6), \quad (8)$$

$$J_{\alpha\beta;\gamma\delta}(x, y) = (\gamma_\mu)_{\alpha\beta} (\gamma_\nu)_{\gamma\delta} J_{\mu\nu}(x, y) \quad (9)$$

and $J_{\mu\nu}$ is expressed as

$$J_{\mu\nu}(x, y) = g^2 \int_C \frac{\partial u_\omega}{\partial x_\mu} du_\varepsilon \int_C \frac{\partial u_{\omega'}}{\partial y_\nu} du_{\varepsilon'} \frac{\text{tr}}{N_c} \langle F_{\varepsilon\omega}(u) F_{\varepsilon'\omega'}(v) \rangle. \quad (10)$$

L_{eff} (8) is written in the contour gauge [15].

It can be identically rewritten in the gauge-invariant form if one substitutes parallel transporters $\Phi(x, x_0), \Phi(y, x_0)$ (identically equal to unity in this gauge) into (8) and (10), multiplying each $\psi(x)$ and $\psi(y)$ respectively and in (10) replacing $F(u)$ by $\Phi(x, u)F(u)\Phi(u, x_0)$ and similarly for $F(v)$.

After that L_{eff} becomes gauge-invariant, but in general contour-dependent, if one keeps only the quartic term (8), and neglects all higher terms. A similar problem occurs in the cluster expansion of Wilson loop, when one keeps only lowest correlators, leading to the (erroneous) surface dependence of the result.

Situation here is the same as with a sum of QCD perturbation series, which depends on the normalization mass μ for any finite number of terms in the series. This unphysical dependence is usually treated by fixing μ at some physically reasonable value μ_0 (of the order of the inverse size of the system).

The integration over contours $D\kappa(C)$ in (6) resolves this difficulty in a similar way. Namely, the partition function Z formally does not depend on contours (since it is integrated over a set of contours) but depends on the weight $D\kappa(C)$ and we choose this weight in such a way, that the contours would generate the string of minimal length between q and \bar{q} . Thus the physical choice of the contour corresponds to the minimization of the meson (baryon) mass over the class of strings, in the same way as the choice of $\mu = \mu_0$ corresponds to the minimization of the dropped higher perturbative terms.

As a practical outcome, we shall keep the integral $D\kappa(C)$ till the end and finally use it to minimize the string between the quarks.

Till this point we have made only one approximation –neglected all field correlators except the Gaussian one. Now one must do another approximation –assume large N_c expansion and keep the lowest term. As was shown in [13] this enables one to replace in (8) the colorless product ${}^f\psi_b(x) {}^g\psi_b^+(y) = \text{tr}({}^f\psi(x)\Phi(x, x_0)\Phi(x_0, y) {}^g\psi^+(y))$ by the quark Green's function

$${}^f\psi_{b\beta}(x) {}^g\psi_{b\gamma}^+(y) \rightarrow \delta_{fg} N_c S_{\beta\gamma}(x, y) \quad (11)$$

and $L_{\text{eff}}^{(4)}$ assumes the form

$$L_{\text{eff}}^{(4)} = -i \int d^4x d^4y {}^f\psi_{a\alpha}^+(x) {}^fM_{\alpha\delta}(x, y) {}^f\psi_{a\delta}(y) \quad (12)$$

where the quark mass operator is

$${}^fM_{\alpha\delta}(x, y) = -J_{\mu\nu}(x, y)(\gamma_\mu {}^fS(x, y)\gamma_\nu)_{\alpha\delta}. \quad (13)$$

From (6) it is evident that fS satisfies to equation

$$(-i\hat{\partial}_x - im_f) {}^fS(x, y) - i \int {}^fM(x, z) d^4z {}^fS(z, y) = \delta^{(4)}(x - y). \quad (14)$$

Equations (13), (14) have been first derived in [13]. From (6) and (12) one can realize that at large N_c the $q\bar{q}$ and $3q$ dynamics is expressed through the quark mass operator (13), which should contain both confinement and CSB.

Indeed, analysis done in [13,14] reveals that confinement is present in the long-distance form of $M(x, y)$, when both distances $|\mathbf{x}|, |\mathbf{y}|$ of light quark from heavy antiquark (placed at $\mathbf{x} = 0$) are large.

We shall do now several simplifying assumptions, to clarify the structure of $M(x, y)$. First of all we take the class of contours C going from any point $x = (x_4, \mathbf{x})$ to the point $(x_4, 0)$ and then to $(-\infty)$ along the x_4 axis. For this class the corresponding gauge was studied in [18]. Secondly, we take the dominant part of $J_{\mu\nu}$ in (13), namely J_{44} , which is proportional to the correlator of color-electric fields, yielding linear confining interaction, and neglect other components $J_{ik}, J_{i4}, J_{4i}, i = 1, 2, 3$, containing magnetic fields and yielding momentum dependent corrections. (It is easy to take into account these contributions in a more detailed analysis).

The correlator $\langle FF \rangle$ in (10) can be expressed through the scalar correlator $D(x)$, defined as [2],

$$\frac{tr g^2}{N_c} \langle F_{\alpha\beta}(u) \Phi(u, v) F_{\gamma\delta}(v) \Phi(v, u) \rangle = D(u - v)(\delta_{\alpha\gamma} \delta_{\beta\delta} - \delta_{\alpha\delta} \delta_{\beta\gamma}) + O(D_1) \quad (15)$$

where the correlator D_1 not contributing to confinement is neglected. As a result one has for M [19]

$${}^fM_{C_{x_4}}(x, y) = {}^fM^{(0)}I + {}^fM^{(i)}\hat{\sigma}_i + {}^fM^{(4)}\gamma_4 + {}^fM_\gamma^{(i)}\gamma_i. \quad (16)$$

Here we have defined

$$\hat{\sigma}_i = \begin{pmatrix} \sigma_i & 0 \\ 0 & \sigma_i \end{pmatrix}. \quad (17)$$

The dominant part of $M, {}^f M^{(0)}$ is linearly growing at large $|\mathbf{x}|, |\mathbf{y}|$ and in the most simple case of Gaussian form of $D(x)$, can be written as

$${}^f M^{(0)}(x, y) = \frac{1}{2T_g\sqrt{\pi}} e^{-\frac{(x_4-y_4)^2}{4T_g^2}} \sigma \left| \frac{\mathbf{x} + \mathbf{y}}{2} \right| \tilde{\delta}^{(3)}(\mathbf{x} - \mathbf{y}) \quad (18)$$

where T_g is the gluon correlation length, and $\tilde{\delta}$ is a smeared δ -function, which can be represented as [19]

$$\tilde{\delta}^{(3)}(\mathbf{x} - \mathbf{y}) \approx \exp\left(-\frac{|\mathbf{x} - \mathbf{y}|^2}{b^2}\right) \left(\frac{1}{b\sqrt{\pi}}\right)^3, \quad b \sim 2T_g. \quad (19)$$

Here T_g is the gluon correlation length, which enters $D(u)$ as $D(u) = D(0)\exp(-\frac{u^2}{4T_g^2})$. We are now in the position to derive $q\bar{q}, 3q$ Green's function, which will be done in the next section.

3. Equations for $q\bar{q}$ Green's function

We start with gauge-invariant definitions of initial (Ψ_{in}) and final (Ψ_{fin}) $q\bar{q}$ states,

$$\begin{aligned} \Psi_{\text{in}}(x, \bar{x}) &\equiv {}^f_1\psi^+(x) \Gamma_{f_1\bar{f}_1}^{\text{in}} \Phi(x, \bar{x}) {}^{\bar{f}_1}\psi(\bar{x}); \\ \Psi_{\text{fin}}(y, \bar{y}) &= {}^f_2\psi^+(y) \Gamma_{f_2\bar{f}_2}^{\text{fin}} \Phi(y, \bar{y}) {}^{\bar{f}_2}\psi(\bar{y}). \end{aligned} \quad (20)$$

The $q\bar{q}$ Green's function is expressed as

$$\begin{aligned} G(x\bar{x}|y\bar{y}) &= \langle \Psi_{\text{in}}(x, \bar{x}) \Psi_{\text{fin}}^+(y, \bar{y}) \rangle = \\ &= \frac{1}{N} \int D\kappa(C) D\psi D\psi^+ \exp\{L + L_{\text{eff}}\} \Psi_{\text{in}} \Psi_{\text{fin}}^+. \end{aligned} \quad (21)$$

One can do integrals over $D\psi D\psi^+$ and assuming as before the large N_c limit, one can neglect the determinant term, yielding

$$\begin{aligned} G(x\bar{x}|y\bar{y}) &= \frac{1}{N} \int D\kappa(C) \{ \text{tr}[S_C^{\bar{f}_1}(\bar{x}, \bar{y}) \Gamma_{f_1\bar{f}_2}^{+\text{fin}} S_C^{f_2}(y, x) \Gamma_{f_2\bar{f}_1}^{\text{in}}] - \\ &\quad - \text{tr}(S_C^{f_1}(x, \bar{x}) \Gamma^{\text{in}}) \text{tr}(S_C^{\bar{f}}(y, \bar{y}) \Gamma^{+\text{fin}}) \} \end{aligned} \quad (22)$$

where we have omitted for simplicity parallel transporters and used notation

$$S_C^f(x, y) = (-i\hat{\partial} - im_f - iM^f(x', y'))_{x,y}^{-1}. \quad (23)$$

At this point we assume that minimization over contours $D\kappa(C)$ in (22) selects one specific contour $\tilde{C}(u)$ for each u (and we find this $\tilde{C}(u)$ in what follows), and choose for simplicity nonsinglet flavour channel. Then the second term in the curly brackets in (22) disappears and we have equation for G

$$\begin{aligned} (-i\hat{\partial}_x - im_1 - iM_{C_1}(x, y))(-i\hat{\partial}_{\bar{x}} - im_2 - iM_{C_2}(\bar{x}, \bar{y}))G(y, \bar{y}|z, \bar{z}) = \\ \delta^{(4)}(x - z)\delta^{(4)}(\bar{x} - \bar{z}) \end{aligned} \quad (24)$$

where we have omitted integrals over $d^4y d^4\bar{y}$ and unit operators $\hat{1} = \delta^{(4)}(x - y)$, $\hat{1} = \delta^{(4)}(\bar{x} - \bar{y})$ as factors of $(-i\hat{\partial}_x - im_1)$ and $(-i\hat{\partial}_{\bar{x}} - im_2)$, respectively. Next steps in treating (24) are standard for the Bethe–Salpeter formalism [20], the only difference is the contour dependence of mass operators M_{C_i} , which we specify as follows. One can choose the class of contours which pass from a point $u = (u_4, \mathbf{u})$ to (u_4, \mathbf{x}_0) and then to $(-\infty, \mathbf{x}_0)$, i.e. parallel to the c.m. motion of the $q\bar{q}$ system. Exact value of \mathbf{x}_0 will be found by minimization of the interaction kernel.

Defining as in [20] the c.m. coordinate X_μ and relative coordinate r_μ .

$$X_\mu = \alpha x_\mu + (1 - \alpha)\bar{x}_\mu, r_\mu = x_\mu - \bar{x}_\mu, \quad (25)$$

where α is an arbitrary parameter and corresponding momenta

$$P_\mu = \frac{1}{i} \frac{\partial}{\partial X_\mu}, \quad p_\mu = \frac{1}{i} \frac{\partial}{\partial r_\mu} \quad (26)$$

one can also fix α to have correspondence with nonrelativistic limit to be $\alpha = m_1(m_1 + m_2)^{-1}$.

Since M_C is invariant with respect to time shifts, one can introduce the total mass of the $q\bar{q}$ system E and relative energy ε and write in the c.m. system an equation for the $q\bar{q}$ wave function

$$\begin{aligned} & \left[\frac{m_1}{m_1 + m_2} E - (m_1\beta_1 + \mathbf{p} \cdot \boldsymbol{\alpha}_1) + \varepsilon - \beta_1 U(\mathbf{x} - \mathbf{x}_0) \right] \\ & \times \left[\frac{m_2}{m_1 + m_2} E - (m_2\beta_2 - \mathbf{p} \cdot \boldsymbol{\alpha}_2) - \varepsilon - \beta_2 U(\mathbf{x} - \mathbf{x}_0) \right] \Psi = 0 \end{aligned} \quad (27)$$

where $U(\mathbf{x} - \mathbf{x}_0)$ is the local limit ($T_g \rightarrow 0$) of the mass operator M_C . For vanishing angular momentum one has

$$U(\mathbf{x} - \mathbf{x}_0) = \sigma |\mathbf{x} - \mathbf{x}_0|. \quad (28)$$

The further analysis and solution of (27) involves as in [21] introduction of positive (+) and negative (-) energy projection operators and corresponding 4 wave functions $\Psi_{\lambda\lambda'}, \lambda, \lambda' = \pm$, for which a system of equations is written similarly to [21].

We shall not follow this route here, however, leaving it for subsequent paper and instead write in the Appendix the Hamiltonian approach to the same problem, which will allow us to calculate spectrum in an approximate way, and discuss it in the Conclusions.

There is still another the 3d form of (27) which obtains in the same way as in [21]. Indeed, writing the time Fourier transform of the Green's function in the form (the sign $\langle \rangle_C$ denotes the integral $D\kappa(C)$)

$$G = \left\langle \frac{1}{(E - E_2 - H_1)(E_2 - H_2)} \right\rangle_C \quad (29)$$

and integrating over dE_2 one arrives as in [21] to the 3d Green's function

$$G(E, \mathbf{r}, \mathbf{r}') = \left\langle \frac{1}{E - H_1 - H_2} \right\rangle_C \quad (30)$$

where

$$H_1 = m_1\beta_1 + \mathbf{p} \cdot \boldsymbol{\alpha}_1 + \beta_1 U(\mathbf{x} - \mathbf{x}_0),$$

$$H_2 = m_2\beta_2 - \mathbf{p} \cdot \boldsymbol{\alpha}_2 + \beta_2 U(\bar{\mathbf{x}} - \mathbf{x}_0). \quad (31)$$

Minimizing interaction over x_0 , one arrives at $x_0 = \frac{\mathbf{x} + \bar{\mathbf{x}}}{2}$, and at the equation for the wave function.

$$\{m_1\beta_1 + m_2\beta_2 + \mathbf{p}(\boldsymbol{\alpha}_1 - \boldsymbol{\alpha}_2) + \beta_1 U(\frac{\mathbf{r}}{2}) + \beta_2 U(-\frac{\mathbf{r}}{2})\}\Psi(\mathbf{r}) = E\Psi(\mathbf{r}). \quad (32)$$

In the nonrelativistic limit this reduces to the usual nonrelativistic quark model,

$$(\frac{\mathbf{p}^2}{2m} + \sigma r)\varphi_n = \varepsilon_n\varphi_n, \quad \varepsilon_n = E_n - m_1 - m_2, \quad m = \frac{m_1 m_2}{m_1 + m_2} \quad (33)$$

4. Equations for the baryon Green's function

Equations for the $3q$ system can be written in the same way as for the $q\bar{q}$ system. We again shall assume large N_c limit in the sense, that $1/N_c$ corrections from $q\bar{q}$ pairs to the quark Green's function and the effective mass can be neglected, but shall write explicit expressions for $N_c = 3$.

The initial and final field operators are

$$\Psi_{in}(x, y, z) = e_{abc}\Gamma^{\alpha\beta\gamma}\psi_{a\alpha}(x, C(x))\psi_{b\beta}(y, C(y))\psi_{c\gamma}(z, C(z)) \quad (34)$$

with the notations: a, b, c , are color indices, α, β, γ are Lorentz bispinor indices and transported quark operators are

$$\psi_{a\alpha}(x, C(x)) = (\Phi_C(x, \bar{x})\psi_\alpha(\bar{x}))_a \quad (35)$$

and the contour $C(x)$ in Φ_C can be arbitrary, but it is convenient to choose it in the same class of contours that is used in $D\kappa(C)$ and in the generalized Fock–Schwinger gauge [15]. $\Gamma^{\alpha\beta\gamma}$ is the Lorentz spinor tensor securing proper baryon quantum numbers. One can also choose other operators, but it does not influence the resulting equations. In (34) we have omitted flavour indices in Γ and $\psi(x, C)$, to be easily restored in final expressions.

Using now the effective Lagrangian (12) valid at large N_c , we obtain for the $3q$ Green's function.

$$G^{(3q)}(x, y, z|x', y', z') = \frac{1}{N} \int D\kappa(C) D\psi D\psi^+ \Psi_{fin}(x', y', z') \Psi_{in}^+(x, y, z) \exp(L_1 + L_{eff}). \quad (36)$$

Integrating out quark degrees of freedom and neglecting determinant at large N_c one has

$$G^{(3q)} = \int D\kappa(C) (e\Gamma)(e'\Gamma') \{S(x, x')S(y, y')S(z, z') + \text{perm}\} \quad (37)$$

where color and bispinor indices are suppressed for simplicity together with parallel transporters in initial and final states.

One can also define unprojected (without Γ, Γ') $3q$ Green's function $G_{un}^{(3q)}$ with 3 initial and 3 final bispinor indices instead of projected by Γ, Γ' quantum numbers of baryon.

Assuming that minimization over contours $D\kappa(C)$ reduces to the single choice of the contours (the single string junction trajectory minimizing the mass of baryon), one can write equation for $G_{\text{un}}^{(3q)}$:

$$(-i\hat{\partial}_x - im_1 - i\hat{M}_1)(-i\hat{\partial}_y - im_2 - i\hat{M}_2)(-i\hat{\partial}_z - im_3 - i\hat{M}_3)G_{\text{un}}^{(3q)} = \delta^{(4)}(x - x')\delta^{(4)}(y - y')\delta^{(4)}(z - z') \quad (38)$$

and e.g. $\hat{M}_1 G \equiv \int M(x, u)G(u, x')d^4u$. One can simplify the form (37) for $G^{(3q)}$ taking into account that $M(x, x')$ actually does not depend on $\frac{x_4 + x'_4}{2}$, and hence the interaction kernel of $G^{(3q)}$ does not depend on relative energies, as in [21]. Similarly to [20,21] one can introduce Fourier transform of $G^{(3q)}$ in time components and take into account energy conservation $E = E_1 + E_2 + E_3$. One obtains

$$G^{(3q)}(E, E_2, E_3) \simeq \int D\kappa(C)(e\Gamma)(e'\Gamma') \times \frac{1}{(E - E_2 - E_3 - H_1)(E_2 - H_2)(E_3 - H_3)} \quad (39)$$

where notations are used

$$H_i = m_i\beta^{(i)} + \mathbf{p}^{(i)}\boldsymbol{\alpha}^{(i)} + \beta^{(i)}M(\mathbf{r}^{(i)} - \mathbf{r}^{(0)}) \quad (40)$$

and we have taken in $M(x, x')$ the limit of small T_g and set of contours in $D\kappa(C)$ passing from the point $\mathbf{r}^{(i)}$ to some (arbitrary) point $\mathbf{r}^{(0)}$.

As in [21] one can now integrate over E_2, E_3 to obtain finally

$$G^{(3q)}(E, \mathbf{r}_i, \mathbf{r}'_i) \simeq \int D\kappa(C)(e\Gamma)(e'\Gamma') \frac{1}{(E - H_1 - H_2 - H_3)}. \quad (41)$$

From (41) one obtains equation for the $3q$ wave function similar to that of $q\bar{q}$ system,

$$(H_1 + H_2 + H_3 - E)\psi(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) = 0 \quad (42)$$

where $\mathbf{r}^{(0)}$ is to be taken at the Torricelli point.

In the nonrelativistic approximation $m_i \gg \sqrt{\sigma}$ one has

$$\sum_{i=1}^3 \left[\frac{(\mathbf{p}^{(i)})^2}{2m_i} + \sigma|\mathbf{r}^{(i)} - \mathbf{r}^{(0)}| \right] \Psi = \varepsilon \Psi \quad \varepsilon = E - \sum m_i \quad (43)$$

5. Inclusion of chiral degrees of freedom

Analysis of the $q\bar{q}$ equations, e.g. (24) and (32), reveals that solutions do not contain Goldstone modes in the chiral limit ($m_1, m_2 \rightarrow 0$). The reason is that the equations have been obtained in the limit of large N_c , while chiral corrections appear in the subleading order, and the coupling constant of pion to the quark is $O(N_c^{-1/2})$. Therefore chiral correction $O(1/N_c)$ to the second term on the r.h.s. of (22) yields contribution of the same order $O(N_c)$ as the first term, since number of color traces in these terms are

different. A similar situation occurs in the vacuum made of instantons surrounded by confining background considered in [22]. There the massive ρ -type pole in the pionic channel produced by the first term in (22) is exactly cancelled by the second term, while the Goldstone pole appears in the chiral correction.

To proceed one can bosonize the quartic Lagrangian (8) in the standard (but nonlocal) way [23]

$$Z = \int D\kappa(C) D\psi d\psi^+ D\omega \exp\{L_1 + L_\omega + L_{\psi\omega}\} \quad (44)$$

where bosonic colorless field $\omega \equiv \omega_{\alpha\beta}^{fg}(x, y)$ enters in $L_\omega, L_{\psi\omega}$ as

$$L_\omega = -\frac{1}{2N_c} \int d^4x d^4y \omega_{\alpha\beta}^{fg}(x, y) J_{\alpha\beta;\gamma\delta}(x, y) \omega_{\gamma\delta}^{fg}(x, y) \quad (45)$$

$$L_{\psi\omega} = -\frac{1}{2N_c} \int d^4x d^4y [\omega_{\alpha\beta}^{fg}(x, y) J_{\alpha\beta;\gamma\delta}(x, y) {}^f\psi_\gamma^+(x) {}^g\psi_\delta(y) + \text{h.c.}]. \quad (46)$$

Integrating out quark degrees of freedom one obtains the effective chiral Lagrangian, $Z \sim \int e^{L_{\text{ch}}} D\omega D\kappa(C)$

$$L_{\text{ch}} = -L_\omega + \text{Trln}(i\hat{\partial} + i\hat{m} + iM + \Delta) \quad (47)$$

where M is the leading $O(N_c^0)$ term (13), while Δ is the chiral correction.

Note that in the limit $T_g \rightarrow 0$, the quark mass operator M becomes local ($\sim \delta^{(4)}(x - y)$) and therefore one can consider in Δ and L_{ch} the usual limit of local chiral field. Parametrizing it as in [24], one has

$$\Delta_\pi = i(e^{i\boldsymbol{\pi}\boldsymbol{\tau}\gamma_5} - 1)M. \quad (48)$$

Proceeding as in [22] one can expand $L_{\text{ch}}(\pi)$ in π_i and obtain

$$L_{\text{ch}}(\pi) = \int \pi_a(k) N(k, k') \pi_a(k') dk dk' \quad (49)$$

where $N(0, 0) = 0$ yielding massless Nambu–Goldstone pions. Similarly to [22] also here the massive pole cancels while the Nambu–Goldstone pole survives. For a detailed discussion the reader is referred to [22] and a subsequent publication.

6. Conclusions

We have obtained above equations for mesons and baryons (27), (32) and (38), (42) respectively which contain all dynamics in the form of nonlocal mass Kernels $M(x, y)$. The latter are solutions of equations (13), (14) obtained earlier in [13] and describing motion of one quark in the field of heavy antiquark. This reduction of $q\bar{q}$ and $3d$ problems to a simpler problem of one quark in the field of static source is possible in the lowest order of $1/N_c$ expansion.

The form of solution for $M(x, y)$ was found quasiclassically in [13, 14] and also numerically in [19] in the thin string limit ($T_g \rightarrow 0$) and for nonrotating string ($L = 0$) reduces to a simple linear potential at large distances.

In the same limit the obtained equations produce reasonable spectrum, see, e.g., equation (A.12) for the $q\bar{q}$ system, which yields results roughly coinciding with the WKB spectrum for spinless Salpeter equation in a case of zero current quark mass [25].

Thus confining dynamics is correctly reproduced by the cited equations. At the same time the exploited form of $M(x, y)$ is applicable at zero angular momentum and does not describe the rotating string, which can be seen in the wrong Regge slope in (A.12). Chiral degrees of freedom which can be taken into account in the same formalism need additional step, which was shortly discussed in Section 5. Here one should take into account the $4q$ form of the effective Lagrangian (8) and do a general (nonlocal) bosonisation procedure, which generates a new operator $\hat{M}(x, y)$, containing bosonic fields, in addition to the scalar-isoscalar component of \hat{M} , dominant at large N_c , which represents the scalar string (and hence chiral symmetry breaking).

The paper has left aside many interesting questions, in particular the exact form of equations for pion Green's function, the coexistence of confining and chiral effects in the dynamics of mesons and baryons, the definition of the constituent quark mass and hadron magnetic moments etc.

All these points are planned for future publications.

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Appendix

Effective Hamiltonian for the $q\bar{q}$ system

Our purpose here is to use the Feynman–Schwinger representation (FSR) [10,11] to express the inverse operator in (23). To this end we represent S_C^f as

$$S_C^f(x, y) = i(\hat{\partial} + m + M)^{-1} = i(-\hat{\partial} + m + M)(-\partial^2 + \delta + M^2)^{-1} \quad (\text{A.1})$$

where $\delta = \hat{\partial}M - M\hat{\partial}$, and with the notation $\tilde{M}^2 \equiv M^2 + \delta$ one has using FSR

$$\begin{aligned} S_C^f(x, y) &= i(-\hat{\partial} + m + M) \int_0^\infty ds e^{-s(-\partial^2 + \tilde{M}^2)} = \\ &= i(-\hat{\partial} + m + M) \int_0^\infty ds (Dz)_{xy} e^{-K - \int_0^s d\tau \tilde{M}^2(z\tau)}. \end{aligned} \quad (\text{A.2})$$

Proceeding now in the same way, as in [7] one obtains

$$\begin{aligned} G(x, \bar{x}|y, \bar{y}) &= \frac{1}{N} \int D\kappa(C) \Gamma^+(-\hat{\partial} + m_{f_1} + M_{f_1}) \Gamma(-\hat{\partial} + m_{\bar{f}_1} + M_{\bar{f}_1}) \\ &\quad \times \int DR D\tau D\mu_1 D\mu_2 e^{-A} \end{aligned} \quad (\text{A.3})$$

where we have used notations

$$\begin{aligned} R_\alpha &= \frac{\mu_1 z_\alpha + \mu_2 \bar{z}_\alpha}{\mu_1 + \mu_2}, \quad r_\alpha = z_\alpha(t) - \bar{z}_\alpha(t), t \equiv z_4, \\ 2\mu_1 &= \frac{dz_4}{d\tau}, 2\mu_2 = \frac{d\bar{z}_4}{d\bar{\tau}}. \end{aligned} \quad (\text{A.4})$$

The Euclidean action A can be written as

$$\begin{aligned} A &= \int_0^T dt \left\{ \frac{m_1^2}{2\mu_1(t)} + \frac{m_2^2}{2\mu_2(t)} + \frac{\mu_+(t)}{2} \dot{R}^2 + \frac{\tilde{\mu}(t) \mathbf{r}^2}{2} + \right. \\ &\quad \left. + \frac{\tilde{M}_c^2(z(t) - x_0)}{2\mu_1} + \frac{\tilde{M}_{\bar{c}}^2(\bar{z}(t) - x_0)}{2\mu_2} \right\}; \end{aligned} \quad (\text{A.5})$$

with the notations $\tilde{\mu} = \frac{\mu_1 \mu_2}{\mu_1 + \mu_2}$, $\mu_+ = \mu_1 + \mu_2$. Integration over $D\kappa(C)$ in (A.3) can be reduced to the minimization with respect to the parameter β , defining $\mathbf{x}_0(\beta) = \mathbf{z}\beta + \bar{\mathbf{z}}(1 - \beta)$, $0 \leq \beta \leq 1$.

At large r one can use the asymptotic form of M , given in (18), then neglecting for the moment the term δ which is constant at large r one has

$$U^2 \equiv \frac{M^2(z - x^0)}{2\mu_1} + \frac{M^2(\bar{z} - x^0)}{2\mu_2} = \sigma^2 r^2 \left(\frac{(1 - \beta)^2}{2\mu_1} + \frac{\beta^2}{2\mu_2} \right) \quad (\text{A.6})$$

Finding the stationary point of U^2 in β at $\beta_0 = \frac{\tilde{\mu}}{\mu_1}$, one finally obtains the minimized U^2 , denoted as $\langle U^2 \rangle$

$$\langle U^2 \rangle = \frac{\sigma^2 r^2}{2\mu_+}. \quad (\text{A.7})$$

Integrating over DR_μ with boundary conditions $R_4(T) = T$ and $R_4(0) = 0$, one has for A

$$A = \int_0^T dt \left\{ \frac{m_1^2}{2\mu_1} + \frac{m_2^2}{2\mu_2} + \frac{\mu_+}{2} + \frac{\tilde{\mu}}{2} \dot{r}^2 + \frac{\sigma^2 r^2}{2\mu_+} \right\}. \quad (\text{A.8})$$

The corresponding Hamiltonian is

$$H = \frac{m_1^2}{2\mu_1} + \frac{m_2^2}{2\mu_2} + \frac{\mu_+}{2} + \frac{\sigma^2 r^2}{2\mu_+} + \frac{\mathbf{p}^2}{2\tilde{\mu}}. \quad (\text{A.9})$$

One can vary (A.9) over μ_1, μ_2 with fixed $\tilde{\mu}$ to obtain for equal masses $m_1 = m_2 = m$

$$H = \frac{\mathbf{p}^2 + m^2}{2\tilde{\mu}} + 2\tilde{\mu} + \frac{\sigma^2 r^2}{2\tilde{\mu}} \quad (\text{A.10})$$

and finally varying over $\tilde{\mu}$ one has

$$H = 2\sqrt{\mathbf{p}^2 + m^2 + \sigma^2 r^2} \quad (\text{A.11})$$

which yields the mass of the $q\bar{q}$ system

$$M^2 = 8\sigma(2n_r + L + \frac{3}{2}) + 4m^2. \quad (\text{A.12})$$

It is remarkable that the spectrum (A.12) is the same (modulo factor of 4) as the WKB spectrum of Dirac equation with the linear confining potential [13].

However the Regge slope of the spectrum (A.12) is $(8\sigma)^{-1}$, which signifies that the rotation of the string connecting q and \bar{q} is not properly taken into account. Here and above we confine ourselves to the zero angular momentum, leaving the problem of rotating string to subsequent publication.

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